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ABSTRACT

In the first part of this paper, we discuss the properties of a second order flexible system of preferences based on demand equations which are linear in income and quadratic in prices for all goods but one. Hausman and Ruud (AER, 1984) introduced a family labor supply model based on this system. We derive explicit expressions for the corresponding direct utility function, conditional demand equations, and concavity conditions in both price/income and quantity space. These results are then used in an empirical static family labor supply model, in which kinked budget constraints and unemployment benefits are taken into account for both spouses. Imposition of concavity is necessary for consistent estimation and the concavity constraint appears to be binding. For females, we find a strongly forward bending labor supply function and a strong impact of the tax system. For males, the own wage elasticity appears to be small and negative. For both spouses, we find small cross-wage elasticities in the unconditional labor supply equations and, correspondingly, small elasticities with respect to the partner's working hours in the conditional labor supply equation.

1. Introduction

The larger part of the recent labor supply literature is devoted to the explanation of female labor supply decisions, thereby addressing the theoretical and econometric problems associated with non-participation, non-linear and non-convex budget sets and stochastic specification (see, for example Heckman (1974), Hausman (1979, 1980, 1985), Moffitt (1986), Arrufat and Zabalza (1986), Blundell and Meghir (1986) and Blundell, Ham and Meghir (1987)). In these papers, male labor supply decisions usually play a role only through a (by assumption exogenous) explanatory variable other household income, which includes male labor earnings.

In this paper we adopt the more general approach of modelling male and female labor supply simultaneously. First of all, there is some evidence that the exogeneity assumption of 'other household income' in female labor supply models is not always tenable; see Smith and Blundell (1986). More importantly, male and female labor supply decisions within a household are likely to be fundamentally interrelated and a full understanding of household's labor supply behavior requires to take this interrelationship into account in setting up the empirical model.

The joint modelling of male and female labor supply creates some specific problems in addition to those encountered in modelling individual labor supply. One of the issues is how to represent the household members' preferences. We will follow the usual approach of assuming that preferences can be represented by a joint household utility function with male leisure, female leisure and total household consumption as arguments. There have been some attempts to develop more general procedures, in which the spouses are allowed to have different preferences and household behavior is the outcome of a game. In order to derive demand functions one then has to specify a certain concept of equilibrium (see Manser and Brown (1980) and McElroy and Horney (1981), for example). However valuable this approach may be from a theoretical point of view, its empirical implementation has not (yet) been very succesful, the main reason being that the available data do usually not allow to identify both utility functions, and to infer which equilibrium concept is appropriate.

A second issue that comes up specifically in modelling joint male and female labor supply is that one usually also has to derive conditional supply equations, i.e. equations that give optimal labor supply of a household member, given a fixed number of hours of labor supply by the partner. For example, if the female partner stops working, the functional form of the male labor supply equation changes from its unconditional to its conditional form (assuming absence of other quantity constraints). For popular flexible functional forms (in the sense of Diewert (1974)), such as the Almost Ideal Demand System and the Indirect Translog, the derivation of conditional demand equations is a cumbersome affair, and closed forms can generally not be obtained. See, for example, Kooreman and Kapteyn (1986).

It appears that at this moment there exist only two flexible forms suited to deal with conditional equations and unconditional equations in a relatively tractable way. The first one is the direct quadratic utility function, which was used for this kind of problem by Wales and Woodland (1980) and later on extensively by Ransom (1987a,1987b). The main disadvantage of this system is the existence of a satiation point, which limits the area in quantity space that can be described by the system. In the case of random preferences, it means that the range of the stochastic parameters has to be restricted. This point will be discussed in slightly more detail in the concluding section. Except for this one complication the direct quadratic utility function is a convenient specification. Yet it seems worthwhile to investigate alternatives, if only for the reason that empirical demand systems are not necessarily described well by the quadratic specification. A second flexible system with reasonable tractability has been introduced by Hausman and Ruud (1984).

Since the properties of the Hausman-Ruud system have not been discussed in the literature extensively, we provide a rather elaborate analysis of the system, including the derivation of the conditional supply equations, the computation of direct utility and the imposition of concavity in wages of the cost function. The need to compute direct utility in an arbitrary point of the choice set may arise if the budget set is non-convex in which case different local utility maxima on convex subsets of the budget set have to be compared. Imposition of concavity in a relevant range of wages is sometimes necessary in empirical applications, as the likelihood function of the model may not be well-defined if concavity is not satisfied.

The practical importance of these issues will be illustrated in an empirical example given in Section 4. In Section 5 we make a brief comparison between the direct qudratic and the Hausman-Ruud. There we also discuss the importance of modelling the labor supply of spouses jointly.

2.1. The model

A household is assumed to maximize a utility function with male leisure, female leisure and total household consumption as its arguments. We assume that the expenditure function in real terms (i.e. expenditures divided by the price of consumption) corresponding to maximization of the utility function under a linear full income constraint is of the Gorman polar form type introduced by Hausman and Ruud (1984):

$$c(w,u) = u \exp(-\beta' w) - \{\vartheta + \delta' w + \frac{1}{2} w' A w\},$$

where $w = (w_m, w_f)'$: the husband's and wife's after tax real wage rates;
u : household utility level;
$A = \begin{bmatrix} \gamma_m & \alpha \\ \alpha & \gamma_f \end{bmatrix}$, $\beta = \begin{bmatrix} \beta_m \\ \beta_f \end{bmatrix}$, $\delta = \begin{bmatrix} \delta_m \\ \delta_f \end{bmatrix}$ and ϑ: parameters.

The corresponding indirect utility function is given by

$$v(w,\mu) = \mu^* \exp(\beta' w),\ \mu^* = \vartheta + \mu + \delta' w + \frac{1}{2} w' A w, \tag{1}$$

where μ denotes the household's real non-labor income. μ^* can be interpreted as the difference (in real terms) between non-labor income and the expenditures needed to reach utility level 0.

Application of Roy's identity yields the following labor supply functions:

$$h^* = \delta + \mu^* \beta + A w, \tag{2}$$

where $h^* = (h_m^*, h_f^*)'$ is the vector of optimal numbers of working hours of husband and wife respectively.

2.2. Concavity

The use of the function given by (1) is limited by the usual regularity conditions on expenditure functions. For this specification, only concavity has to be considered, i.e. the matrix of second order partial derivatives of the expenditure function must be negative semi-definite and of rank 2; homogeneity and monotonicity with respect to u are satisfied automatically.
It is easy to show, that concavity is equivalent to

$$B \equiv \mu^*\beta\beta' - A \qquad \text{is negative definite}^{1)} \tag{3}$$

From now on we assume that the matrix A is non-singular.
Note that, if $\beta'A^{-1}\beta \neq 0$, a <u>necessary</u> condition for concavity is given by

$$\mu^* < \{\beta'A^{-1}\beta\}^{-1} \tag{3'}$$

If $\beta \neq 0$ and $\beta'A^{-1}\beta=0$, then B is negative definite for no value of μ^*. This case is excluded from now on. In the special case that A is positive definite, it is easy to prove that (3') is not only necessary but also sufficient for (3). (See, for a proof of a more general result, Bekker (1986)).

The application of duality theory strongly hinges on the concavity condition; without this property, there is no utility maximizing problem behind the labor supply equations. Therefore, (3) must hold for all relevant (w,μ), including shadow wages and corresponding virtual incomes.

2.3. <u>The Direct Utility function</u>

Non-convexity of the budget set makes it necessary to compare the values of the direct utility function in different points. We shall derive the direct utility function by calculating the utility level in some arbitrary point (h_m, h_f, y), where y is the household's consumption (or income):

$$y = \mu + w_m h_m + w_f h_f \tag{4}$$

Let k be the vector $h-\delta$, where $h = (h_m, h_f)'$. Given (h_m, h_f, y), we first seek (shadow-)wages w and corresponding non-labor income μ satisfying

$$k = \mu^*\beta + Aw \tag{5}$$

$$\mu^* = \mu + \vartheta + w'\delta + \frac{1}{2}w'Aw \tag{6}$$

$$\mu = y - w'h \tag{7}$$

---

1) B is just the Hessian of the expenditure function. Since the expenditure function is defined in terms of real wage rates, the ususal condition that the Hessian of the expenditure function is negative semi-definite, is replaced by (3).

Inserting the solution (w,μ) from (5), (6) and (7) in the indirect utility function (1) then yields the utility level at (h_m, h_f, y).

Equations (5) through (7) yield, after substituting (7) into (6):

$$w - A^{-1}k = -\mu^* A^{-1}\beta \tag{8}$$

$$\mu^* = \frac{1}{2}(w - A^{-1}k)'A(w - A^{-1}k) - \frac{1}{2}k'A^{-1}k + y + \vartheta \tag{9}$$

Substituting (8) into (9) yields a quadratic equation in μ^*:

$$\frac{1}{2}\mu^{*2}\beta'A^{-1}\beta - \mu^* - \frac{1}{2}k'A^{-1}k + y + \vartheta = 0 \tag{10}$$

and if μ^* is known, w can be found from (8):

$$w = A^{-1}(k - \mu^*\beta) \tag{11}$$

Thus (w,μ) can be determined iff (10) has a real solution, i.e. iff

$$1 + \beta'A^{-1}\beta\{k'A^{-1}k - 2(y + \vartheta)\} \geq 0 \tag{12}$$

a solution (w,μ) is only feasible if it satisfies concavity condition (3). Obviously, if $\beta=0$, the solution of (10) and (11) is unique and it satisfies (3) if and only if A is positive definite. If $\beta\neq0$ and (12) holds, then (10) and (11) yield (at most) two solutions (w,μ^*) and only the smallest of the two satisfies the necessary condition (3'):

$$\mu^* = (\beta'A^{-1}\beta)^{-1} - \{(\beta'A^{-1}\beta)^{-2} + (\beta'A^{-1}\beta)^{-1}[k'A^{-1}k - 2(y + \vartheta)]\}^{1/2} \tag{13a}$$

$$w = A^{-1}(k - \mu^*\beta) \tag{13b}$$

$$\mu = y - w'h \tag{13c}$$

If this solution satisfies (3), then it is feasible and the utility level is given by

$$U(h_m, h_f, y) = V(w_m, w_f, \mu) = \mu^* \exp(\beta'w) \tag{14}$$

The reader should be aware of the relation between invertibility (i.e. the question whether (w_m, w_f, μ) can be solved as a function of (h_m, h_f, y)) and concavity (i.e. well-behavior of the direct or indirect utility function). As usual in dually specified systems, the concavity condition involves (shadow-) wages and it can therefore only be checked in (h_m, h_f, y)-space if invertibility is guaranteed. In the special case of a positive definite matrix A, a specific property of the specification used is the fact that, if (w_m, w_f, μ) can be found, then exactly one solution satisfies the concavity conditions (i.e.: "invertibility guarantees concavity").

2.4. Rationed labor supply

In this subsection, we derive rationed labor supply functions, i.e. labor supply for one individual if - for some reason - the partner's number of working hours is fixed. This means, that the household maximizes utility, taking into account some binding constraint on one of the three goods.

Rationed supply curves can be determined using shadow-wages and shadow-income (see Neary & Roberts (1980)).[2] We derive the female's rationed labor supply h_f for given h_m, actual real wage rates w_m and w_f and real non-labor income μ. (The male's rationed labor supply can be derived in exactly the same way)

We search for a shadow wage rate $\bar{w}_m$ and corresponding $\bar{\mu}$, such that

$$h_m = \beta_m \bar{\mu}^* + \gamma_m \bar{w}_m + \alpha w_f + \delta_m \tag{15a}$$

$$\bar{\mu} + h_m \bar{w}_m = \mu + h_m w_m \tag{15b}$$

$$\bar{\mu}^* = \bar{\mu} + \theta + \delta_f w_f + \delta_m \bar{w}_m + \frac{1}{2}(\gamma_f w_f^2 + \gamma_m \bar{w}_m^2) + \alpha w_f \bar{w}_m. \tag{15c}$$

If a feasible solution $(\bar{w}_m, \bar{\mu})$ (with corresponding $\bar{\mu}^*$) is found, optimal female labor supply is given by

$$h_f = \beta_f \bar{\mu}^* + \gamma_f w_f + \alpha \bar{w}_m + \delta_f \tag{16}$$

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2) Rationed supply curves can alternatively be determined using first order conditions for maximization of the direct utility function, which is explicitly derived in section 2.3, subject to the budget constraint and the ration levels.

The system (15) implies

$$a_2\bar{w}_m^2 + a_1\bar{w}_m + a_0 = 0 \tag{17}$$

where

$$a_0 = -h_m + \beta_m\{\mu + \vartheta + h_m w_m + \delta_f w_f + \tfrac{1}{2}\gamma_f w_f^2\} + \alpha w_f + \delta_m,$$

$$a_1 = \gamma_m + \beta_m\{-h_m + \delta_m + \alpha w_f\},$$

$$a_2 = \tfrac{1}{2}\beta_m\gamma_m.$$

If (17) has no real solution, no shadow wage can be found and h_f cannot be determined. Equation (17) has a real solution iff

$$D \equiv \beta_m^2(-h_m + \delta_m + \alpha w_f)^2 + \gamma_m^2 - 2\beta_m^2\gamma_m\{h_m w_m + \mu + \vartheta + \delta_f w_f + \tfrac{1}{2}\gamma_f w_f^2\} \geq 0 \tag{18}$$

If $\bar{w}_m$ is found, then $\bar{\mu}$, $\bar{\mu}^*$ and h_f follow immediately from (15) and (16). The solution is feasible iff it satisfies concavity condition (3).

We focus on the "regular" case, i.e. $\beta_m\gamma_m \neq 0$. If (18) holds, the solutions for $\bar{w}_m$ are given by

$$\bar{w}_m = -\beta_m^{-1} + (h_m - \delta_m - \alpha w_f)/\gamma_f \pm (\beta_m\gamma_m)^{-1}\sqrt{D}.$$

The corresponding value of $\bar{\mu}^*$ is

$$\bar{\mu}^* = \beta_m^{-2}\gamma_m \mp \beta_m^{-2}\sqrt{D} \tag{19}$$

Since the matrix $\beta_m^{-2}\gamma_m\beta\beta'$-A is indefinite or semi-definite and the matrix $\beta\beta'$ is positive semi-definite, it is easy to see that only one solution can be feasible:

$$\bar{w}_m = -\beta_m^{-1} + (h_m - \delta_m - \alpha w_f)/\gamma_f + (\beta_m\gamma_m)^{-1}\sqrt{D}. \tag{20}$$

Note that, even in the special case of a positive definite matrix A, this solution is not necessarily feasible; condition (3) should always be checked. Thus, the relation between "partial invertibility" and concavity is different from the relation between "full invertibility" and concavity, which was discussed in Section 2.3.

In this section we derived the conditional female labor supply function $h_f(w_f, h_m, \mu+w_m h_m)$ corresponding to household preferences given by (14). The result is a closed form expression for h_f. Lundberg (1988) follows a different strategy: She starts with conditional demand functions in some convenient form and does not discuss the issue whether it is possible to find a household utility function corresponding to these equations. Our approach has the advantage that, since a closed form expression of the indirect utility function is available, it is easy to check whether the underlying system of preferences satisfies regularity properties (e.g. concavity) and allows for the use of non-convex budget sets.

3. Applications

The rationed labor supply functions derived in Section 2.4 can be applied in several situations. The most common example is the nonnegativity constraint for females. If this restriction is binding, the husband's labor supply function should be replaced by a rationed labor supply function, as described in Section 2.4. The same argument holds for the analysis of implications of mandatory reduction of the working week, as proposed by some Western European governments, on labor supply of individuals for whose partner this reduction is binding.

A similar situation arises if individual budget sets are piecewise linear and convex (see e.g., Blomquist (1983) and Hausman (1979)), as in the case where spouses file separately and the tax system is progressive and piecewise linear. The household budget set in this case is depicted in Figure 1. In The Netherlands, this budget set is a reasonable approximation for families not entitled to unemployment benefits. If, for example, the optimal number of the husband's working hours is at a kink, then female labor supply is not given by (2) but by the conditional labor supply function given in Section 2.4.

If the budget set is non-convex, comparison of values of the direct utility function is necessary to determine the optimum, as is described in Section 2.3. Unemployment benefits or fixed costs of working are common phenomena causing such non-convexities, in particular at zero hours of work.

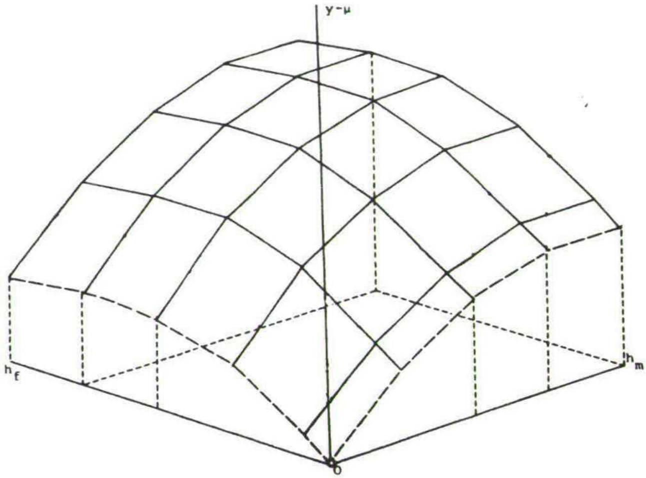

Figure 1. The household budget set if individual budget sets are piecewise linear and convex

Apart from constraints arising from the shape of the budget set, restrictions may stem from demand side factors or institutional constraints on the labor market. Particularly in The Netherlands, actual hours are not only determined by labor supply decisions of the household, but also strongly depend on institutional constraints, such as agreements between unions and employers, and demand side factors. Most jobs in The Netherlands are 40 hours a week jobs, with a fixed number of holidays and strong limitations on working overtime. Especially in the manufacturing sector, there are only a limited number of part-time jobs. Possibilities to work a non-standard number of hours are rare. It therefore seems unrealistic to treat actual hours as if they were chosen freely by the membersof the family.

This is one of the reasons why several recent Dutch labor market surveys do not only contain information on actual hours worked, but also on preferred hours, i.e. the number of hours someone would like to work under a given scenario. Although the description of such a hypothetical scenario is never complete, the formulation of the questions in the most recent surveys seems to leave practically no room for misinterpretation. Preferred hours are provided by respondents in a ceteris paribus context, i.e. it is assumed that the partner does not change his or her actual number of working hours. This way of questioning implies, that preferred hours in the data set are to be interpreted as optimal hours, conditional on the fact that the actual number of hours worked by the partner is fixed.[3] Thus, a conditional labor supply equation as described in Section 2.4 is needed to explain preferred hours.

Some further explanation may be useful at this point. Of course, preferred hours are not very interesting by themselves from an economist's point of view; it is actual hours that we want to study eventually. But, due to institutional constraints and demand side factors, preferred hours appear to be a better reflection of the household's preferences than actual hours. Thus, certainly in The Netherlands, it is preferred hours we should use to reveal preferences. In a later stage, the information on family preferences should be used to construct a labor market model, in which actual hours are linked to preferences and institutional constraints and demand side factors.

4. An empirical example

In this section, we present an application of the model studied in Section 2. A similar model, estimated for a different data set, can be found in Kapteyn & Woittiez (1988). In that paper, some of the results derived here have been used. For the rest, the Kapteyn & Woittiez paper concentrates on different issues, particularly habit formation and preference interdependence. In our model preferred hours of husband and wife are the endogenous variables, for reasons discussed in Section 3.

4.1. Specification of the model

Since each individual provides his or her preferred number of working hours, taking the partner's actual labor supply as given, only conditional labor supply functions are relevant. From the individual's point of view the household budget-set is therefore only two-dimensional. In Figures 2a and 2b, approximate budget sets are drawn for a female, whose husband works h_m hours a week.

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3) A typical wording of the survey question asking for preferred hours is: "How many hours would you like to work if you could choose freely and if your hourly after tax wage rate remains as it is now? Assume that other family members do not change their number of working hours"

Figure 2a relates to a working female. The budget curve consists of 11 income tax brackets. Non-convexities do not arise, because working people who quit voluntarily are not entitled to unemployment benefits. The optimal number of hours in this case can be found by computing conditional labor supply for each of the brackets, as described by Hausman (1979) and Blomquist (1983), since fixing male labor supply has reduced the dimension of the problem. The optimum h_f^* can be in the interior of one of the brackets or at one of the kinks, as in the situation drawn. It may also be negative.

If a female is unemployed and receives benefits $c_f>0$, the budget set is non-convex. We assume, that the individual looses all benefits at the moment she works slightly more than zero hours. This assumption is in itself incorrect, but since the marginal tax rate on increased earnings for someone on unemployment compensation is close to 100%, so that a choice of a number of hours corresponding with an earned income below the unemployment benefit level is unlikely, it appears to be rather harmless.
The optimum in this case (see Figure 2b) can be either 0 hours or h_f^*, depending on the fact whether the utility level $U_0=U(h_m,0,\mu+w_mh_m+c_f)$ exceeds $U_1=U(h_m,\max(0,h_f^*),\mu+w_mh_m+w_f\max(0,h_f^*))$ or not. (In Figure 2b, the former is the case).

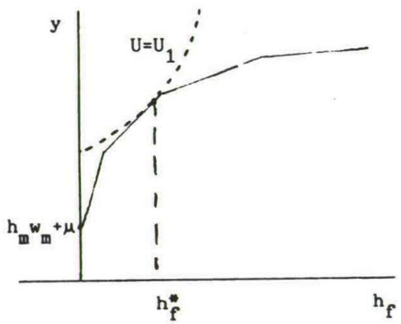

Figure 2a. The budget set; no unemployment benefits

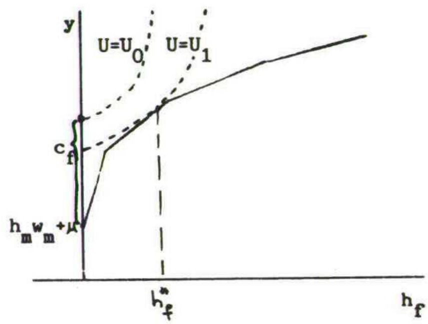

Figure 2b. The budget set with unemployment benefits

The stochastic specification in utility rationing models is a delicate problem, even in the case of a convex budget set (see, e.g., Kooreman and Kapteyn (1986)). In these types of models it is important to distinguish between different sources of random errors, i.e. measurement errors, optimization errors and random preferences.

Preference variation across households in our model could be incorporated by allowing the parameters δ_m and δ_f to depend upon household characteristics:

$$\delta_i = \sum_{j=1}^{K} x_j \delta_{ij} + \varepsilon_i \qquad (i=m,f) \tag{21}$$

where x_j (j=1,...,K) are observed characteristics (including a constant term) and ε_i is a random variable representing unobserved sources of preference variation. This corresponds to translating, see McElroy (1987)

Random δ's, however, lead to random shadow wages and a complicated likelihood function. Moreover, the lack of global concavity, as discussed in Section 2.2, implies that it is necessary to truncate the distribution of the ε's in some rather intricate way. It is easy to see that conditions like (3') or (12) imply that the ε's have to lie in a polyhedron and it is hard to find a tractable distribution which allows for such a kind of truncation. Although we do recognize the importance of a stochastic specification that allows for random preference variation, the ensuing complications make this an issue beyond the scope of this paper.

Our stochastic specification is "ad hoc" in the sense, that it only allows for optimization (or measurement) errors. We add normally distributed error terms to the conditional labor supply functions.
Thus, for a female not receiving an unemployment compensation, we have

$$h_f^p = \max\{0, h_f^* + \varepsilon_f\}$$

where h_f^p is the observed preferred number of working hours and h_f^* is the optimal choice given the budget constraint.[4)]

If a female does receive an unemployment compensation, we only know whether she is seriously looking for a job or not. The optimization error η_f is incorporated as an error in the "regime choice":

$$v = u_1 - u_0 + \eta_f.$$

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3) For individuals who work less than 15 hours a week, it is only known whether preferred hours exceed actual hours or not. It is straightforward to take this into account, considering h^p as a latent variable.

If $v>0$, the female wants to work; if $v<0$, she is not seriously looking for a job. Male preferred labor supply is treated in exactly the same way.

The vector of error terms $(\varepsilon_f, \varepsilon_m, \eta_m, \eta_f)'$ is assumed to follow a multinormal distribution with mean zero and covariance matrix

$$\begin{bmatrix} \sigma_m^2 & . & . & . \\ \rho\sigma_m\sigma_f & \sigma_f^2 & . & . \\ * & 0 & \sigma_v^2 & . \\ 0 & * & * & \sigma_v^2 \end{bmatrix}$$

An asterisk indicates that the variance does not appear in the likelihood function, so that it cannot be estimated. Because of the small number of people in the sample receiving an unemployment benefit, we impose $\mathrm{cov}(\varepsilon_m,\eta_f) = \mathrm{cov}(\varepsilon_f,\eta_m) = 0$, and $\mathrm{var}(\eta_m) = \mathrm{var}(\eta_f)$.

4.2. Data and estimation results

The data used stem from a labor mobility survey conducted in The Netherlands in 1982 by the Institute of Social Research of Tilburg University jointly with the Netherlands Central Bureau of Statistics. The data set has been used by various researchers in The Netherlands for studies on labor supply, labor mobility, and income distribution. The survey was held among a random sample of Dutch households with at least one household member between 16 and 65 years of age. In each household, all members between 16 and 65 years have been interviewed. The information collected pertains to incomes, hours worked, desired hours, search behavior, demographics, etc. Non-response is equal to 35.7 %. Comparison with population characteristics shows that the survey is fairly representative of the population from which it was drawn, although students and unemployed people appear to be somewhat underrepresented. Altogether the survey comprises 2677 persons in 1299 households. The analysis here is restricted to families with at least two adults. Also, self-employed, students, and disabled people are omitted from the sample. As a result, in the estimation of the household labor supply model, data on 520 households were used. Some sample statistics are given in Table 1.

The before tax wage rates in Table 1 are predicted wages on the basis of a wage equation with log(age), log(age)-squared and education as predictors. For males and females separate wage equations have been estimated, using Heckman's two-stage procedure (Heckman (1979)).

Table 1. Sample Statistics

		mean	standard deviation	min	max	number of obs.
males	preferred hours	37.60	6.62	15	70	489
	actual hours (all males)	39.77	11.78	0	70	520
	actual hours (working males only)	42.29	6.38	20	70	489
	before tax wage rate	14.97	7.66	4.98	55.87	520
	after tax wage rate	11.74	3.72	4.95	29.48	520
	unemployment benefit (recipients only)	357.70	96.40	228.99	644.38	26
females	preferred hours	24.49	8.52	8	50	133
	actual hours (all females)	8.44	13.24	0	42	510
	actual hours (working females only)	22.62	12.20	2	42	194
	before tax wage rate	14.18	5.72	3.41	23.26	520
	after tax wage rate	12.71	5.12	3.06	21.82	520
	unemployment benefits (recipients only)	132.33	71.59	50.83	184.11	3
non-labor household income		80.62	122.54	0	927.41	520
log (family size)		1.200	0.349	0.693	2.303	520
dummy children < 6 years old		0.317	0.466	0	1	520

Explanation:

hours: working hours per week

wage rates: in Dfl. per hour worked

benefits: in Dfl. per week

non-labor income: in Dfl. per week, not including unemployment benefits.

Table 2. Estimation results

Parameter	Estimate	Standard error[5]
α	0.88×10^{-3}	0.13×10^{-2}
β_m	-0.20×10^{-2}	0.10×10^{-2}
β_f	-0.47×10^{-3}	0.47×10^{-3}
γ_m	0.86×10^{-2}	0.93×10^{-2}
γ_f	0.47	0.20
δ_{m0}	32.3	2.2
δ_{f0}	24.0	4.1
δ_{m1}	3.9	1.0
δ_{f1}	-24.0	3.5
δ_{m2}	-0.40	0.82
δ_{f2}	-13.9	2.7
σ_m	6.7	0.12
σ_f	19.3	1.7
σ_v	21.4×10^{10}	32.5×10^{11}
ρ	-0.21	0.07
Θ	-390.18	- [6]

Explanation: the parameters δ_m and δ_f have been made dependent upon additional exogenous variables as follows:

$$\delta_i = \sum_{j=0}^{2} X_j \delta_{ij} \qquad (i=m,f)$$

$$X_0 = 1$$

$$X_1 = \log(\text{family size})$$

$$X_2 = \begin{cases} 1 & \text{if there are children in the family younger than six} \\ 0 & \text{otherwise} \end{cases}.$$

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5) Covariance matrix of the parameter estimates is estimated as outer product.

6) The estimate of ϑ attains its upper bound (due to the imposition of concavity) so no standard error could be computed.

The model has been estimated by means of maximum likelihood.[7] To impose concavity of the cost function in wages in a relevant region of the (h_m,h_f,y)-space, the parameter ϑ has been restricted, i.e. an upper bound in terms of other parameters in the model has been set to ϑ, such that concavity is guaranteed in all data points[8]; it turns out that this restriction is binding. It should be noted that testing of the restriction is impossible, since the likelihood is not well-defined under the alternative. This problem is discussed in more detail in Van Soest et al. (1988).

Table 2 presents the parameter estimates. β_m ("the male non-labor income effect") is significantly negative and γ_f (representing the largest part of the female own wage effect) is significantly positive, whereas β_f, α and γ_m do not differ significantly from zero. β_m and β_f have the expected sign, indicating that leisure is a normal good. The variables concerning family composition play a significant role in the female hours equation but not in the male hours equation. A direct economic interpretation for the parameters other than β_m and β_f is hard to give. The economic meaning of the estimates is brought out more clearly by graphs and elasticities.

In Figures 3 a through d family labor supply functions are drawn for a family without children as a function of before tax wage rates. In each case the remaining variables are set at their sample means. We distinguish between "short run" (the partner is rationed at a certain number of hours) and "long run" (the partner is not rationed) labor supply functions. In each of the four figures two short run labor supply functions are drawn: one for the case where the actual number of hours worked by the partner equals the sample mean ($\bar{h}_f$ =

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7) A table with likelihood contributions is available on request. The likelihood contributions vary according to whether one or two spouses are participating, whether or not the budget set is convex, whether or not preferred hours are zero, etc.

8) For a positive definite matrix A, concavity is equivalent to (3'). Substituting (19) and (18) into (3') yields

$$\vartheta \leq -\{\mu+h_m w_m+\delta_f w_f+\tfrac{1}{2}\gamma_f w_f^2\} + \tfrac{1}{2}\gamma_m^{-1}(\delta_m-h_m+\alpha w_f)^2-\tfrac{1}{2}\gamma_m^{-1}\beta_m^2(\beta' A^{-1}\beta)^{-2}+(\beta' A^{-1}\beta)^{-1}.$$

This restriction - and a similar one for male labor supply - has been imposed for all sample observations.

22.62 or $\bar{h}_m$ = 42.29) and one for the case where the partner does not work at all.

Figure 3a shows a backward bending male labor supply function implying that the negative income effect dominates the positive own wage effect. Figures 3b and 3c reveal the expected negative relationship between one's preferred number of hours and the partner's wage rate, but the effects are small. Figure 3d shows that female labor supply is forward bending. The own wage impact is much larger for the wife than for the husband. Figure 3d also reveals the working of the tax system. The piece-wise linear progressive tax system leads to jig-sawed responses of preferred hours to the own before-tax wage rate. The reason for this is that each time an individual is at a kink in the budget constraint, she wants to stay there if change the before-tax wage rate changes a little bit. To stay at a kink with an increasing before-tax wage rate entails a reduction of work effort. The downward sloping parts in Figure 3d are hence hyperbolas. The same kind of non-differentiabilities is in principle also present in Figure 3a, but in this case the hyperbola parts are so small that the drawing cannot reveal them. This is caused by the very small male own wage effect.

The difference in own wage elasticities is borne out by Figure 4 in which some indifference curves are depicted, using the results of Section 2.3. Figure 4a shows a few indifference curves upon which the husband's decision is based if his wife works $\bar{h}_f$ = 22.62 hours; it is easy to see that a change in the male wage rate only has a small impact on the optimal number of male working hours. In Figure 4b, where the wife's indifference curves are drawn if the husband works $\bar{h}_m$ = 42.29 hours a week, the (own) wage impact is much larger (Note the difference in scale of both figures).

Similar figures could be drawn for different family compositions. The main difference would be a strong downward shift in all female labor supply functions (due to the negative estimates for δ_{f1} and δ_{f2}, the parameters that represent the impact of family size and the presence of children younger than six respectively on the wife's labor supply). As a result (predicted) preferred hours of the wife are then only non-zero for very high female wage rates.

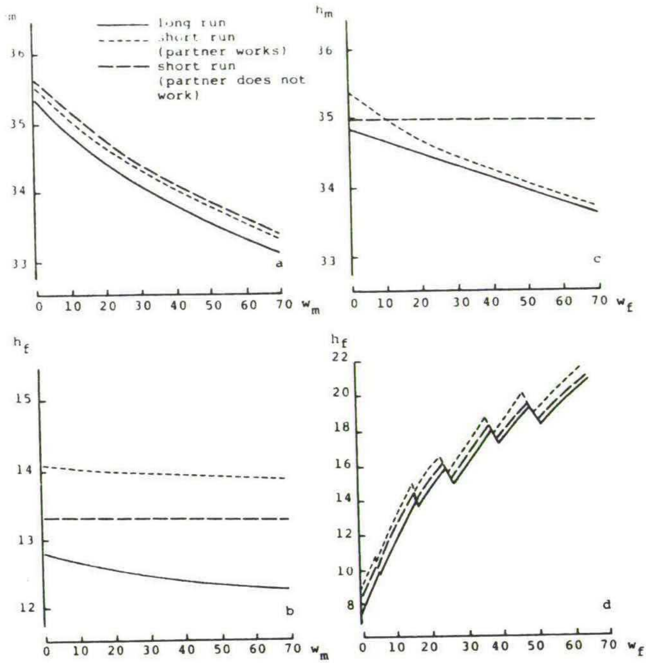

Figure 3. Preferred hours as a function of before tax hourly wage rates for a couple without children.

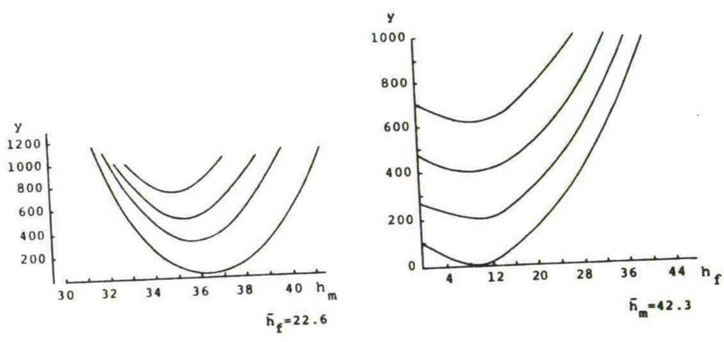

Figure 4. Some indifference curves for a family without children if the number of working hours of one spouse are fixed.

5. Conclusions

Consistent modelling of household labor supply under different regimes (i.e. for different kinks and corners) requires the use of shadow prices if one wants to work with specifications that are given in dual form. Unfortunately, most of the known flexible forms have the undesirable property that shadow prices cannot be found in closed form, except for some special cases. It appears that the only reasonably flexible forms which allow for the computation of shadow wages in closed form are the direct quadratic utility function and the Hausman-Ruud specification. Of course, knowing shadow prices for any point amounts to knowing the direct utility function. Indeed the first thing accomplished in this paper is the derivation of the direct utility function corresponding to the Hausman-Ruud specification. Secondly, the application of rationing theory requires that the system considered satisfies the Slutsky conditions in all data points. Hence we have imposed concavity conditions for all data points in the empirical example considered.

The obvious advantage of the joint modelling of labor supply of both spouses in a family is that once the preferences are known (have been estimated) we are able to predict household behavior under different regimes. This is impossible, for example, if female labor supply were modelled without taking into account the interaction of husband and wife. In the latter case it would be impossible to say what would happen if the male changes the number of hours worked, or changes from a state of unemployment to a state of employment. It is true that the pictures in Figure 4 suggest that actually not much will happen in such a case, but of course one can only know that after an empirical analysis in which the interaction of both spouses has been taken into account properly.

A drawback of the Hausman-Ruud specification might seem to be that it is difficult to allow for random preferences in a utility consistent way. At first sight the direct quadratic utility function does not suffer from such a problem. Ransom (1987b) presents a specification with random errors and provides conditions under which the ensuing model is coherent. The conditions are easy to impose and estimation of the model by ML is rather straightforward. It turns out however that for certain values of the random preferences the bliss point of the direct quadratic utility function is inside the budget constraint, and in such a case the demand equations do not represent a utility maximum. We have shown elsewhere (Van Soest, Kooreman, and Kapteyn (1988)), that the restrictions on the random preferences which are required to prevent this from happening are identical to the restrictions that have to be imposed in the Hausman-Ruud system on the random preferences to guarantee a well-behaved system. Therefore, there are no compelling a priori reasons to prefer one system or the other; we have two reasonably tractable flexible sytems available which can be used for the analysis of household labor supply in the presence of kinks and corners, and the choice between them in each case should be based on the data at hand.

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